# Application of Fuzzy Logic for Autonomous Bay Parking of Automobiles

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**Abstract:** In this paper, we investigate the control problem of autonomous bay parking system. We choose a referenced parking lot and define a suitable parking spot based on some measurements at various places. A kinetic model is set up for the convenience of analysis and simulation. The pose of the car during the parking procedure can be determined by the initial pose, the backward speed, and the steering angle of the wheel. Then, both a fuzzy speed controller and a fuzzy steering controller are designed for the bay parking. Finally, simulation results show the effectiveness of our designed controllers.

Keywords: Fuzzy logic, bay parking, kinetic model, simulation, autonomous vehicle control.

# 1 Introduction

As the number of automobiles is increasing, the streets and the parking lots are becoming more crowded, which makes the car parking more difficult, even for experienced drivers. An increasing number of car accidents is caused by collisions while parking cars. According to a report in China, parking is the main source of vehicle accidents or automobile-induced troubles<sup>[1]</sup>.

Generally, there are three main aspects that make parking difficult<sup>[1, 2]</sup>: First, as the drivers are seated inside while parking cars, they may not see the space around the vehicles well. The information obtained from the rear mirrors is not perfect, due to the existence of blind zones. Second, parking is a complicated procedure that depends on the driving experience, skill, and drivers' reaction. Third, the parking area, the environmental differences between the day-time and night-time, and the effect from other automobiles around also add to the complexity of the parking task.

Due to the above problems and reasons, researchers and engineers have developed autonomous parking systems or parking assist systems to alleviate the driving burden and enhance the safety. Reversing radar is a kind of parking assistant device that can inform the driver about surrounding obstacles in audio or visual forms. The radar system helps the driver get some information from the rear side. Hiramatsu<sup>[3]</sup> developed a new parking assist system based on a rear view camera. The idea is to allow the driver to follow the system by voice message rather than visual guide information through the screen of an information LCD for safety reasons. The guide algorithm minimizes the longitudinal parking space under "two turns" parking maneuvers and helps avoid collisions. However, this approach is just a passive method (not an autonomous way), since the reversing system is operated by the driver.

An autonomous parking system can park vehicles with-

out the assistance of drivers. It consists of a parking spot detection module, and an autonomous parking algorithm. The system first explores an effective parking spot that is larger than the minimal parking space. Meanwhile, the orientation and position of the vehicle relative to the parking space are obtained. Weis et al.<sup>[4]</sup> proposed a method for camera vision-based parking spot detection that is characterized by specific situations, and realized by image processing on the incoming camera images. Vision-based approaches were studied in [5–9]. Ultrasonic sensors were also employed to detect parking lots, see [10–12].

The autonomous parking algorithm has attracted attention in the past decades. Early in 1957, Dubins<sup>[13]</sup> investigated the path curve from an initial point to an end point. However, the movements in [13] are either forward or backward, i.e., mixed motion is not permitted. Reeds and  $\text{Shep}^{[14]}$  enriched the theory in [13], where an optimal algorithm was proposed to calculate the path for a car that moves both forward and backward. When an advanced intelligent control method was developed, it was usually applied to the parking problem to show the effectiveness due to the nonlinearity in the motion. Nguyen and Widrow<sup>[15]</sup> utilized neural network theory to solve the parking problem. Kong and Kosko<sup>[16]</sup> compared the neural network method and fuzzy-logic-based method for the same problem. Fuzzy logic obtained a better control precision in [16]. The objective in [13–16] is to control a vehicle from one position to another position. However, the constraints of the parking lot are not considered.

Gómez-Bravo et al.<sup>[17]</sup> dealt with the problem of parallel and diagonal parking of wheeled vehicles by using fuzzy logic, based on the analysis of collision avoidance. By employing genetic algorithm's learning ability, parameters for the developed fuzzy logic controllers were determined effectively in [10]. A time-varying fuzzy sliding mode controller (TFSC) was developed for repeated scheduling parameters

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and tracking local paths for parallel parking in [18]. An enhanced fuzzy logic controller was designed in [11]. For an autonomous parking system, the method of evolutionary functional testing was used to derive a proposed area criterion that results in a faster convergence of the optimization in [19]. The case of parking of articulated vehicles was addressed in [20, 21]. However, the research subjects were wheeled robots or car-like vehicles. This is essentially different when the developed algorithms are directly adopted into an automobile, due to the compliance and characteristics of the mechanical steering system.

Autonomous parking systems have been regarded as the next gadget in automobile industry. They have two working patterns: general parallel parking and bay  $parking^{[1, 2]}$ . In this paper, we focus on developing a maneuvering algorithm for bay parking of Jetta by properly tuning a fuzzy controller. The design of fuzzy logic controller mainly depends on experimental data. In order to obtain better data, a data collection experiment, in which the automobile is driven by an experienced driver, is carried out. The distance from the car to the origin of the coordinate and the yaw rate are collected, when the automobile is at different initial positions and orientation angles. By using a novel technique, the relationship between X coordinate, Y coordinate, orientation angle, and time is established. Since the backward speed and steering angle vary during the parking process, two fuzzy logic controllers are designed. One fuzzy speed controller is first designed by experience. The other controller is for steering control, and the controller is designed via an adaptive network-based fuzzy inference system (ANFIS). The processed data is used to build the ANFIS for the bay parking in anfisedit, which is a Toolbox of Matlab. Finally, the simulation results show the effectiveness of the proposed two-stage fuzzy controller for the bay parking pattern.

# 2 Bay parking system and preliminaries

## 2.1 Overall process

As shown in Fig. 1, there are four main steps in the bay parking pattern. First, the car is driven close to the parking spot. Then, the system is turned on, and the system detects a suitable parking space. When the parking lot is detected, that is, the lot is larger than a minimal effective lot, and the coordinate is established, the car stops in a recommended area (details on the recommended area can be seen in the work in [1]). Finally, the car is moved to the detected lot by using the designed two-stage fuzzy logic speed and steering controller. In this paper, we focus on studying the control algorithm and designing the controller. Note that the control algorithm is based on the lot detection. Hence, we assume that the lot is detected, and the coordinate system is established.



Fig. 1 The overall process of bay parking pattern

## 2.2 Dimension parameters

In this paper, we investigate the bay parking algorithm for a Jetta car, which is simplified as a rectangle. The main car dimension parameters of our interest are shown in Fig. 2. The values for the parameters are listed in Table 1.



Table 1 Values for the dimension parameters

Dimension parameter	Value (mm)
Length $(L_l)$	4 428
Width $(L_b)$	1660
Wheel base $(L_a)$	2471
Front overhang $(L_{fa2h})$	825
Rear overhang $(L_{ra2t})$	1070
Front tread $(L_{fw})$	1429
Rear tread $(L_{rw})$	1422
Distance between two king pins $(L_{pp})$	1 329
Distance between the rear wheel to the right side $(L_{rr2r})$	119

The angle  $\theta_0$ , as shown in Fig.2, can be computed as follows:

$$\theta_0 = 90^\circ + \arctan\left(2 \times \frac{L_{ra2t}}{L_b}\right) =$$

$$90^\circ + \arctan\left(2 \times \frac{1\,070}{1\,660}\right) = 143^\circ.$$
(1)

#### 2.3 Referenced and suitable parking lot

Generally, different automobiles do not have identical sizes. Similarly, parking lots in different places are not the same either. For the self-adaptation of the control algorithm, it is natural to find the referenced parking lot by considering the dimensions of the car and the sizes of the lots. Any lot, larger than the referenced parking lot, is called a suitable parking lot. Now, we have two statements:

1) If a car can be parked automatically into a referenced parking lot, with the same initial pose and initial position, it can also be parked into a suitable parking lot successfully.

2) For a smaller referenced parking lot, it is more difficult to design the control algorithm. However, if our algorithm can park the car into a smaller space, then it can park the car into a larger one too.

As shown in Table 2, most of the lots' lengths are above  $5\,000$  mm, and the widths are more than  $2\,300$  mm. Due to the above fact, we choose the referenced parking lot as  $4\,871$  mm  $\times 2\,260$  mm, as shown in Fig. 3. The length of the parking lot is chosen as 1.1 times of  $L_I$ . The definition for a suitable parking is that the length of the parking lot is no less than  $4\,871$  mm, and the width is no less than  $2\,260$  mm.

Table 2 Dimensions of parking lots in various places

Places	Length (mm)	Width (mm)
A (College)	4920	2 322
B (Campus of Jilin University)	5922	2530
C (Campus of Jilin University)	5903	3235
D (Bank)	5450	2721
E (Restaurant)	5268	2415
F (Station)	5000	2506



Fig. 3 The referenced parking lot

#### 2.4 Criteria for a successful parking

When we assess the performance of the control algorithm, we consider the following facts:

1) The automobile should not collide with other cars in the neighboring spots.

2) Finally, the automobile should be within the parking lot. The shortest distance from the car to the boundary of the lot should not exceed the limit of the sensor.

To satisfy the above requirements, we propose the following criteria for successful bay parking:

1) During the parking process, the projection of the car on the ground cannot touch the left, rear, and right boundaries.

2) In the final pose, the car should be within the lot. Moreover, each point in the projection of the car on the ground must be 100 mm away from the boundary of the lot. Generally, the sensing limit of an ultrasonic sensor is not greater than 100 mm.

#### 2.5 Kinetic model for the system

During the process of parking, the speed of the car is very low. Thus, the dynamics of the tires can be neglected.

Suppose the parking lot is detected and the target parking lot is suitable. Set up a coordinate system, as shown in Fig. 4. In the system,  $(X_f, Y_f)$  is the coordinate of the center point of the front axis,  $(X_r, Y_r)$  denote the coordinate at the center point of the rear axis,  $\theta$  represents the orientation angle,  $\phi$  is the steering angle at the center point of the front axis, and  $V_f$  is the speed at the center point of the front axis. Since the speed is low, and there is no slide at the rear shaft, the speed at  $(X_r, Y_r)$  along the rear shaft is zero, that is,

$$\dot{X}_r \sin(\theta) - \dot{Y}_r \cos(\theta) = 0.$$
<sup>(2)</sup>



Fig. 4 Target parking lot and coordinate system

In terms of geometry, the relationship between  $(X_f, Y_f)$ and  $(X_r, Y_r)$  is

$$\begin{cases} X_r = X_f - L_a \cos(\theta) \\ Y_r = Y_f - L_a \sin(\theta). \end{cases}$$
(3)

Taking the derivative of  $X_r$  and  $Y_r$ , we obtain

$$\begin{aligned}
\dot{X}_r &= \dot{X}_f + \dot{\theta} L_a \sin(\theta) \\
\dot{Y}_r &= \dot{Y}_f - \dot{\theta} L_a \cos(\theta).
\end{aligned}$$
(4)

The relationship between  $(X_{d1}, Y_{d1})$  and  $(X_r, Y_r)$  can be

expressed as

$$\begin{cases} X_{d1} = X_r - L_{r2lb} \cos(180^\circ - \theta_0 - \theta) = \\ X_r + L_{r2lb} \cos(\theta_0 + \theta) \\ Y_{d1} = Y_f + L_{r2lb} \cos(180^\circ - \theta_0 - \theta) = \\ Y_r + L_{r2lb} \sin(\theta_0 + \theta). \end{cases}$$
(5)

The derivative of (5) is

$$\begin{cases} \dot{X}_{d1} = \dot{X}_r - \dot{\theta} L_{r2lb} \sin(\theta_0 + \theta) \\ \dot{Y}_{d1} = \dot{Y}_r + \dot{\theta} L_{r2lb} \cos(\theta_0 + \theta). \end{cases}$$
(6)

Since the angle from  $V_f$  to the negative direction of X axis is  $\theta - \phi$ , we have

$$\begin{cases} \dot{X}_f = -V_f \cos(\theta - \phi) \\ \dot{Y}_f = -V_f \sin(\theta - \phi). \end{cases}$$
(7)

Substituting (4) into (2), we obtain

$$\dot{X}_f \sin(\theta) - \dot{Y}_f \cos(\theta) + \dot{\theta} L_a = 0.$$
(8)

It follows from (7) and (8) that

$$\dot{\theta} = V_f \frac{\sin(\phi)}{L_a}.$$
(9)

By substituting (8) and (9) into (4), we get

$$\begin{cases} \dot{X}_r = -V_f \cos(\theta) \cos(\phi) \\ \dot{Y}_r = -V_f \sin(\theta) \cos(\phi). \end{cases}$$
(10)

Taking the integral for (10) along the time t, we obtain

$$\begin{cases} X_r = -\int_0^t V_f \cos(\theta) \cos(\phi) dt \\ Y_r = -\int_0^t V_f \sin(\theta) \cos(\phi) dt. \end{cases}$$
(11)

At each time instance, the coordinate of  $(X_{d1}, Y_{d1})$  is

$$\begin{cases} X_{d1} = -\int_0^t V_f \cos(\theta) \cos(\phi) dt + L_{r2lb} \cos(\theta_0 + \theta) \\ Y_{d1} = -\int_0^t V_f \sin(\theta) \cos(\phi) dt + L_{r2lb} \sin(\theta_0 + \theta). \end{cases}$$
(12)

It is noted that, in the above analysis, we have a variable  $\phi$  that is an equivalent steering angle at the center point of the front shaft. However, this angle cannot be measured. According to Ackerman steering<sup>[22]</sup> and our experiment (three angular transducers: one for the left tire, one for the right tire, and the other one for the steering wheel), we derive a simple equation:

$$\lambda = 16.6\phi \tag{13}$$

where  $\lambda$  is the steering angle of the wheel. If we have the coordinate of  $(X_{d1}, Y_{d1})$  and the orientation angle  $\theta$ , we can modify the pose and the position of the car. Using the relationship between the steering angles of the wheel and  $\phi$ , we can control the steering of the car. Hence, (12), (13), and (9) can be used to represent the kinetic model during the parking process.

## 3 Controller design

In Section 2, we established the kinetic model. It was shown that the position and the pose of the car can be determined by the velocity  $V_f$  and the steering angle  $\lambda$ . In this section, we will design two fuzzy logic controllers.

#### 3.1 Fuzzy logic speed controller

Our control strategy for the backward speed is 1) at the beginning, gradually increase the speed from 0 km/hto 3 km/h, 2) keep the speed at 3 km/h, and 3) reduce the speed until 0 km/h when the car is successfully parked at the desired spot. This is a typical fuzzy process. There are two inputs for the fuzzy speed controller: One is  $Y_{d1}$ , and the other one is  $L_u$ , which is the mileage at  $(X_r, Y_r)$  from the initial time to the current time. The diagram of the controller is shown in Fig. 5 with 5 IF-THEN rules.



Fig. 5 Diagram of the speed fuzzy controller

The membership functions for  $L_u$ ,  $Y_{d1}$ , and  $V_f$  are illustrated in Figs. 6–8, respectively. The fuzzy rules for the bay parking are chosen as

- 1) IF  $Y_{d1}$  is S, THEN  $V_f$  is S;
- 2) IF  $Y_{d1}$  is M, THEN  $Y_f$  is M;
- 3) IF  $Y_{d1}$  is B AND  $L_u$  is not S, THEN  $V_f$  is B;
- 4) IF  $L_u$  is S AND  $Y_{d1}$  is B, THEN  $V_f$  is S;
- 5) IF  $L_u$  is M, THEN  $V_f$  is M.







Here, S, M, and B represent small, medium, and big, respectively. By using the T-S inference model, we obtain the relationship between the inputs and the output, as shown in Fig. 9.



Fig. 9 Relationship between the inputs and the output

**Remark 1.** From our experiment, the speed of 3 km/h is easy to maintain and control. Moreover, it is at low speed, under which the vehicle dynamics can be neglected. The main idea for the fuzzy speed control is to ensure that the speed is small when the vehicle is close to origin  $(Y_{d1})$ , and speed can be medium or large when the distance is small (In this case, the vehicle is at the beginning of the backing up).

#### 3.2 Fuzzy steering controller

There are two inputs for the fuzzy steering controller:  $\theta$  and  $X_{d1}$ . The output is the steering angle of the wheel. Fig. 10 shows the diagram of the steering fuzzy controller.

However,  $\theta$  and  $X_{d1}$  cannot be measured directly. In our work, we propose an algorithm to derive  $\theta$ ,  $X_{d1}$ , and  $Y_{d1}$  in terms of the steering angle of the wheel  $\lambda$ , the distance of the center point of the front shaft S, and the yaw velocity  $V_{\theta}$ . It is worth pointing out that  $\lambda$ , S, and  $V_{\theta}$  are measured from the initial point to the stop point. The algorithm is given as follows:



Fig. 10 Diagram of the steering fuzzy controller

#### Algorithm 1.

**Step 1.** Smooth the data of S, and get the function between S and the time.

**Step 2.** Take the derivative of the smoothed S, and obtain the absolute velocity. Smooth the absolute velocity, and get the function between the absolute velocity and the time.

**Step 3.** Smooth the yaw velocity  $V_{\theta}$ .

**Step 4.** Integrate the smoothed  $V_{\theta}$  with respect to the time. Then, we get the variation of the orientation angle. The orientation angle during the parking is derived as

$$\theta = \theta_1 + |\theta_2| \tag{14}$$

where  $\theta_1$  is the initial orientation angle, and  $\theta_2$  is the change of the orientation angle during the parking.

Step 5. Smooth the steering angle of the wheel.

**Step 6.** The coordinate change of the center point of the rear shaft is obtained by

$$\begin{cases} \Delta X_r = \sum V_f \cos(\theta) \cos(\phi) \Delta t \\ \Delta Y_r = \sum V_f \sin(\theta) \cos(\phi) \Delta t. \end{cases}$$
(15)

**Step 7.** The coordinate of the point D during the parking is derived as

$$\begin{cases} X_{d1} = \max(\Delta X_r) - (\Delta X_r + L_{r2lb}\cos(\theta + \theta_0)) + 170\\ Y_{d1} = \max(\Delta Y_r) - (\Delta Y_r + L_{r2lb}\sin(\theta + \theta_0)) + 168. \end{cases}$$
(16)

When we obtain extensive dataset, we employ the AN-FIS Toolbox of Matlab to generate the membership functions for  $X_{d1}$  and the orientation angle  $\theta$ . The generated membership functions are shown in Figs. 11–12.



Fig. 12 Generated membership function for the orientation angle

# 4 Simulations

In this section, we assume that the car has different initial conditions including the positions and the poses. We set up the kinetic model that is built in Section 2 in Simulink. Meanwhile, the fuzzy logic controllers are implemented in Simulink model too. The trajectories of the car for various initial conditions appear in Figs. 13–15, which show that the car is parked successfully.



Fig. 13 Trajectory when the initial orientation angle is  $5^{\circ}$ 



Fig. 14  $\,$  Trajectory when the initial orientation angle is  $0^\circ$ 



Fig. 15 Trajectory when the initial orientation angle is  $-5^{\circ}$ 

It is necessary to point out that if the orientation angle is larger, and the initial position in the X-direction is closer to the detected lot, the parking problem is more challenging. As the linear matrix inequalities are playing an important role in systems and control<sup>[23-33]</sup>, in the future, we will employ the techniques of linear matrix inequalities and preview control to solve the bay-parking problem.

## 5 Conclusions

We presented the application of fuzzy control for autonomous bay parking of automobiles. We investigated the available parking space dimensions in Changchun, PRC, and chose a referenced parking lot. A kinetic model was set up for the case, when the backward or forward speed is low, and there is no slide in the rear shaft. By assuming that the parking lot is detected, two fuzzy controllers were designed. One is used to control the speed and the other one is for steering control. The effectiveness under different initial poses and positions is illustrated in the simulation. Our further research is to implement the control logic into a microcontroller and design an actuator so that the control algorithm can be tested in a real automobile.

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